# EXPERIMENTS WITH CYCLIC CURVES I
## AN INNOVATIVE TEACHING METHODOLOGY IN GEOMETRIC DESIGN

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## ABSTRACT

*"Experiments with cyclic curves I" is the result of specific interdisciplinary experiments with Graphic Representation. The main purpose is the development of a systemic, holistic and prospective approach related to Design teaching and research, especially Geometrical Design and Descriptive Geometry, attempting to devise an innovative methodology.*

***Key Words:*** *Geometrical Design, Projective Design, Discontinuities, Theory of Singularity, Theory of Catastrophe, Ergonomic Design, Systemic Approach and four-dimensionality.*

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## INTRODUCTION

In every field which requires graphic representation, the teaching of geometrical design and descriptive geometry is usually developed acoording to traditional methodologies, making use of traditional instruments. Computing opens new ways to education and research in graphic representation, making the teaching of such subjects special and distinct, aiming at keeping abreast of the scientific and technological advances.

The sparse bibliography on methodological innovations in the teaching of Geometrical Design and Descriptive Geometry has been the reason of these experiments.

Ergonomy, which is fundamental to projects, either in Architecture or Industrial Design, has been used as a starting point.

As any other research, effort we have started by questioning reality, namely by specifyng.

- What are the problems in geometrical design and descriptive geometry necessary to represent graphically the ergonomic problems in Industrial Design and Architecture projects?

- What is the teaching methodology in graphic representation suitable to develop the students' systemic view and their interest for research and new findings?

## DEVELOPMENT

We have contacted researchers in graphic representation who were developing studies on Ergonomy.

Prof. Paulo Kawauchi from FAAC-UNESP-BAURU, was just starting a research on the movement of a worker's arm while performing a certain task, having as reference, an article on the subject: "Industrial Workplace Layout".

These American authors show that the normal and the maximum areas used in an industrial task may be determined by parametrical equations. According to the authors, the curves obtained from these equations are EPICYCLOID.

To go from the visual perception to a reading of the non-verbal, we have started by analysing the projection of the movement of arm in true size. For this first step, we examined the possible movements of a person's arm in a horizontal, lateral and vertical planes.

## GENERAL PROBLEM

We have started by analysing the Traditional Teaching Methodology of Geometrical Design and Descriptive Geometry, which treat the specificities of Plane Geometry  and Space Geometry in an isolated and Cartesian manner. Evidently, Geometrical Design and Descriptive Geometry Traditional Teaching Methodology follows the criteria of the industrial age.

a - Standardization of teaching programs, methodology, examinations, etc. It is known which subjects related to Descriptive Geometry, Geometrical Design, etc. The exams are also similar to a certain group of students.

b - Synchronization of syllabuses and subjects in undergraduate courses.

c - Graduagte Studies for Teachers in Geometrical Design, Descriptive Geometry, Technical Design, Ergonomy, etc.

d - Centralization of the know-how required by the subjects within the bounds of the teacher's knowledge. It is the teacher himself or the teaching department that determines what, how, when and how much to teach about Geometrical Design or Descriptive Geometry.

e - Concentration of knowlegde on Geometrical Design and Descriptive Geometry found in books which were published at the beginning of the century and bave been copied by many different authors, with practically no innovation.

f - Maximization of knowledge required by Geometrical Design and Descriptive Geometry, which at times are not effectivelly used for practical purposes.

## ON THE CYCLIC CURVES

We could have any subject on Geometric Design as an example. For that experiment, we analysed the teaching of cyclic curve drawing, since this was the subject which mattered to the demonstration of the published article "Industrial Workplace Layout".

1. Cyclic curves are only taught in Geometrical Design in their three procedures: Cycloid, Epicycloid and Hipocycloid (simple, long, shortened)

2. Some points are assessed: the line quality, the precision of the construction, the theoretical demonstration by means of Euclidian Geometry.

3. The development of abstract thought is an end as an end in itself.

4. It is not demanded the estabilishiment of the relation of these curves and time, i.e., and four-dimensionality.

## PROBLEMS

On the development of the animated approach: How can can one analyse and prove the kind of line that an ordinary movement of the upper arm describes in space?

On the objectives: Devise an appropriate innovative methodology which could relate to the two questions asked at the beginning of this research, which could motivate Design Teaching System and could create an innovative graphic language.

## FIRST OBSERVATIONS

We have started by observing and selecting the position of arm, forearm and hands which throw themselves in true dimension in the horizontal, frontal and lateral planes.

Positions on the horizontal plane
(view from above)

Positions on the frontal plane
(view from the front)

Positions on the profile plane
(view from one side)

## IT HAS BEEN DETERMINED

a. The maximum movements of arm, forearm and hands on each of the afore mentioned projection planes.

b. The maximum angles of each movement of the arm, forearm and hands on each projection plane.

c. The proportions of the average lengths of the arm, forearm, and hands analysed in different people.

d. The average lenths of the arm, forearm and hands, analysed in different people.

## THE DEVELOPMENT OF THE SYSTEMIC APPROACH

How could it be demonstrated that such records are similarly and contiguously correlated subjects, developed in Geometrical Design and Descriptive Geometry?

## SIMILARITY ANALYSIS

a. The points of flexibility of the human arms are circumference centers: the shoulder, the elbow and the wrist.

b. The lengths of the arm, forearm and hand are radii of three circurnferences.

## MOVEMENT VARIATIONS

**VARIATION 1**

With no flexibility: in the wrist or elbow.
With maximum flexibility in the shoulder.
Shoulder: center of an arc of circumference whose radius is equal to the addition of the lengths of the arm, forearm and hand.

**VARIATION 2**

With no flexibility in the wrist or shoulder.
With flexibility in the elbow.
Elbow: center of an arc of circumference whose radius is equal to the lengths of the forearm + hand.

**VARIATION 3**

With no flexibility in the shoulder or elbow.
With flexibility in the wrist.
Wrist: center of an arc of circumference whose radius is equal to the length of the hand.

**VARIATION 4**

With no flexibility in the wrist.
With flexibility in the shoulder and elbow.
Shoulder and elbow: are aligned and therefore are the centers of two arcs of circumference which have internal tangents, and the tangent point is the extremity of the middle finger.

**VARIATION 5**

With flexibility in the shoulder, elbow and wrist.
Shoulder, elbow and wrist: are aligned and therefore are the centers of three arcs of circumference which have internal tangents, having as the tangent point the extremity of the middle finger.

## CONTIGUOUS ANALYSIS

**ANALYSIS 1**

If the maximum shoulder, elbow and wrist flexions occur in sequence, in different time intervals, we will have a curved line made up of three synchronic arcs, denominated by spiral of three centers.

**ANALYSIS 2**

If the maximum shoulder, elbow and wrist flexions occur within the same period, we will have a continuous movement of the points in space.

During this continuous movement, the extremity of the middle finger describes a segment of curved line with infinite centers.

During this continuous movement, the shoulder determines only one point in space, while all the points which are between the elbow and the shoulder, and also the elbow itself describe arcs ot circumference concentric to a fixed center, which is the shoulder.

During this continuous movement, all the points which are between the elbow and the wrist, and also the wrist itself describe curved lines concentric to infinite centers which are the infinite positions of the elbow in space.

During this continuous movement, all the points which are between the wrist and the extremity of the middle finger, and also this extremity, describe curved lines concentric to infinite centers which are the infinite positions of the wrist in space.

## CONCLUSIONS

1. *On teaching methodology and research in Geometrical Design and Descriptive Geometry.*

The teaching of Geometrical Design as a purpose in itself, on the bidimensional space, eliminates the possibility of analysing other metric relationships of the four-dimensional space.

This experiment has made us think about the infinite Geometrical Design Teaching Methodologies that could be researched on, taking into account four-dimensionality.

2. *On the denomination of the curve as being epicycloid, according to the authors of the published article "Industrial Workplace Layout".*

At first we concluded, by similarity, that the movements of the parts of a man's upper arm describe segments of cyclic curve named hipocycloid, since the circumferences are internally tangent and their center describe trajectories which are arcs of concentric circumferences.

If we analyse separately the movements described by the points in space, relating only two circumferences at a time, the graphic representation in the bidimensional space presents two hipocycloid segments.

Segment 1 – The circumference with the center in the shoulder and the radius = arm + forearm + hand as being the directrix circumference of hipocycloid.
The circumference with the center in the elbow and radius in the forearm + hand being the generatrix circumference.

Segment 2 – The circumference with center in the elbow and radius forearm + hand, being the directrix circumference. The circumference with center in the wrist and radius equivalent to the length of the hand, being the generatrix circumference.

In plane geometry, the hipocycloid is generated by a fixed point in a circumference which spins, tangenting internally to another without sliding.

Therefore, a whole turn of the generatrix circumference determines in the directrix circumference, an arc whose circumference is equal to the perimeter of the generatrix circumference.

While carrying out the research, we arrived at the conclusion that:

The previously conceived conclusion would be true if the man were a robot, programmed to perform precise movements.

Man belongs in Nature and, therefore, he isn't an artificial Geometric system, but a natural organic system.

Perhaps, due to that, the continuous movement within the same period makes the points describe different speed trajectories, relating the trajectory lengths to the inequality of the maximum are perimeters of the directrix circumferences and of the generatrix circumferences analysed.

The chart of this curved line shows us a progressive discontinuity on the space traversed as well as on the velocity applied at each space fraction, and also the time spent by each point belonging to the curve.

**New objectives of graphic representation**

For this kind of research, the design teacher must be kept up-to-date.

We guess the reading of the non-verbal, together with recent information about the Theory of Singularity developed by Hassler Whitney, and mainly, the most recent studies about the Theory of

Catastrophe would be the best path for researchers.

The precise graphic representation has always been used in Architecture, Engineering, Industrial Design, etc. based on Euclidean Geometry on bi and tridimensional spaces.

We could not simply call this trajectory by a the cyclic curve designated hipocycloid, having as references the circumferences named directrix and generatrix, just relating them to their positions in the Euclidean theory.

For this kind of graphic representation, we would need a new type of research related to the Theory of Catastrophe, where Geometrical Design would be the graphic construction of the descontinuous, and not only the theorems related to Euclidian Geometry.

Graphic representation would be four-dimensional and not only bi or tridimensional.

It would be needed to devise specific computer software to chart singularities, bifurcations and elasticity.

The study of arithmetical progression and continuous and descontinuous geometry is a subject of fundamental interest to graphic representation in the age of computing.

**Graphic Representations**

In order to represent graphically the point trajectories in space, continuously within the same period, each arc of circumference whose centers are in the shoulder, elbow and wrist, were divided into 6 equal parts.

The possibility to calculate and graphically represent this continuous movement was enhanced by making use of computer graphics.

During this experiment, which is related to Geometrical Design Constructions and the teaching of graphic representation of orthogonal projections used in Descriptive Geometry and Technical Drawing, the most for colored illustrations, procedure was the two dimensions.

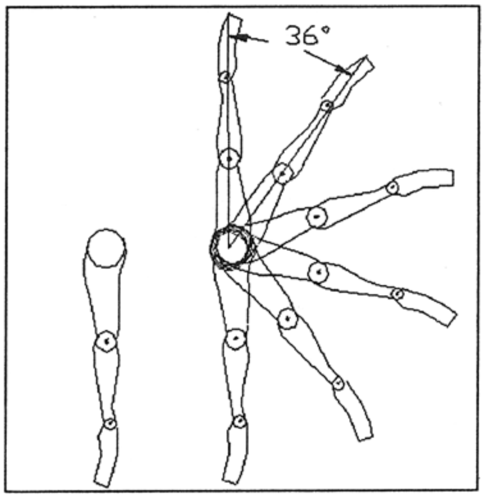



**Fig. 1 - Lateral Plane:** Projection of a maximum movement in an arc of circumference whose center is the shoulder and radius = arm+ forearm+ hand

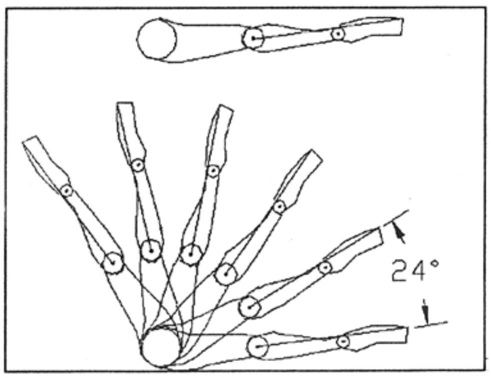



**Fig. 2 - Horizontal Plane:** Projection of a maximum movement in an arc of circumference whose center is the shoulder and radius = arm+ forearm+ hand

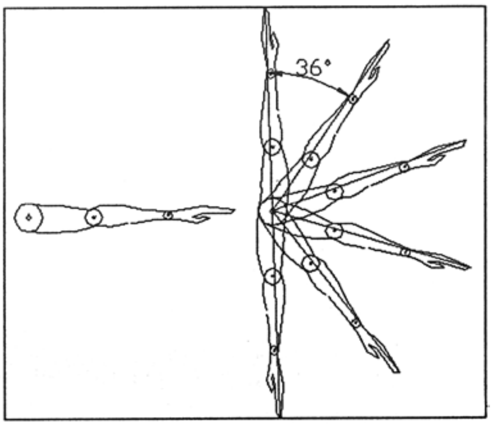



**Fig. 3 - Vertical Plane** Projection of a maximum movement in an arc of circumference whose center is the shoulder and radius = arm+ forearm+ hand

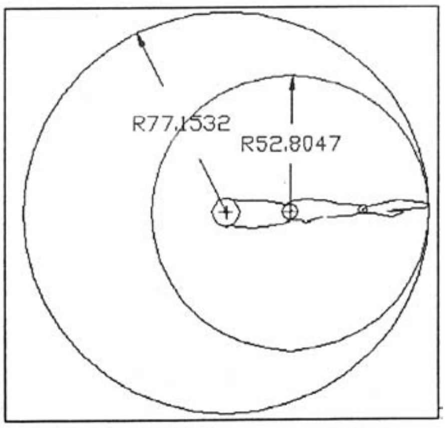



**Fig. 6 - Possible centers of flexibillity:** shoulder and elbow

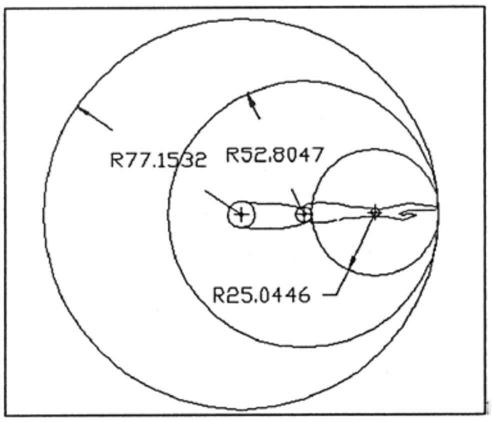



**Fig. 4 - Possible centers of flexibility:** shoulder, elbow and wrist.

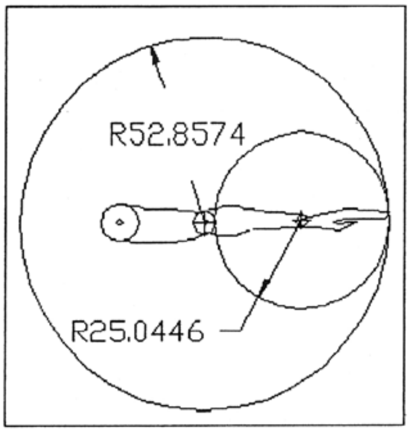



**Fig. 7 - Possible centers of flexibility:** shoulder, elbow and wrist.

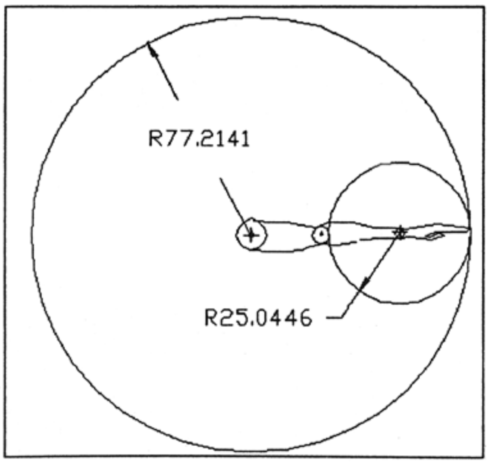



**Fig. 5 - Possible centers of flexibilyty:** shoulder and wrist.

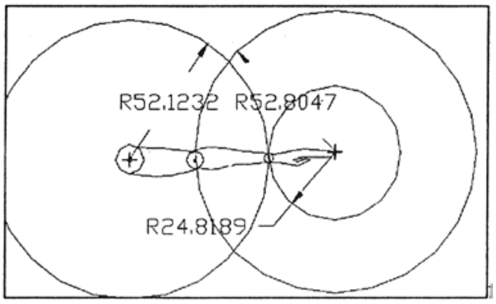



**Fig. 8 - Centers of flexibility with impossible radii.**

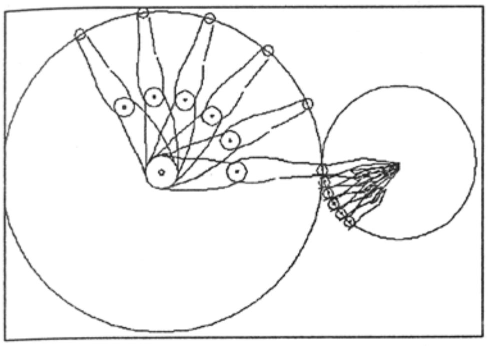


**Fig. 9 - Impossible movements:** Ares of circumference tangent externally

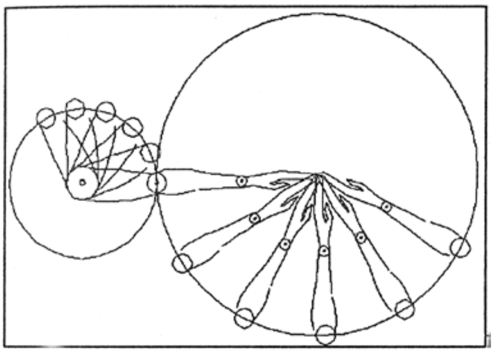


**Fig. 10 - Impossible movements:** Ares of circumference tangent externally

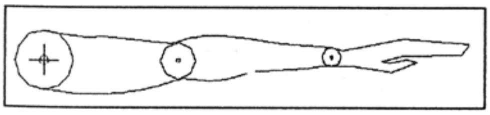


**Fig. 11 - Maximum movement of the hand:** Arcs of circunferencewhose center is the wrist and the radius is the length from the wrist to the extremity of the middle finger.

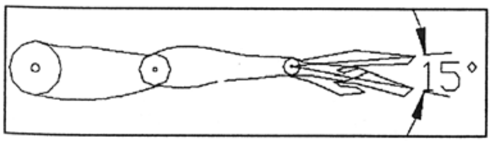
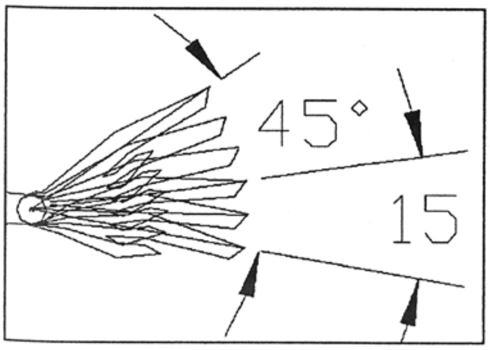



**Fig. 12 - Maximum movement of the hand:** Arcs of circunferencewhose center is the wrist and the radius is the length from the wrist to the extremity of the middle finger.



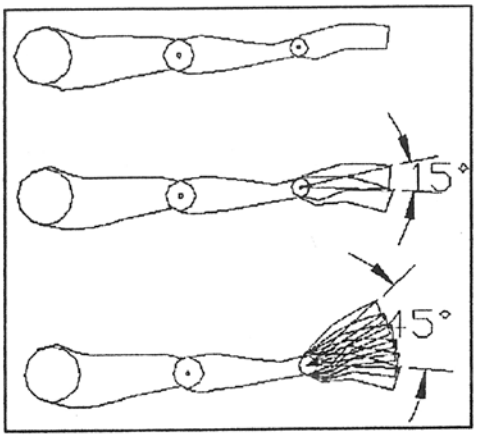




**Fig. 13 e 14 - Maximum movement of the hand:** Arcs of circunferencewhose center is the wrist and the radius is the length from the wrist to the extremity of the middle finger.

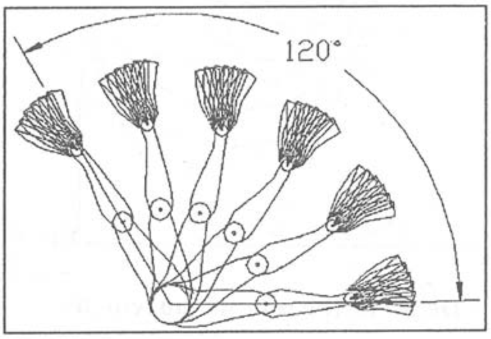



**Fig. 15 - Horizontal Plane:** two maximum movement.

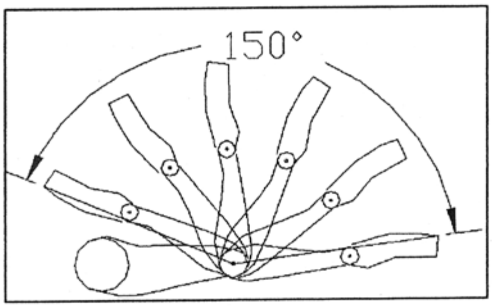



**Fig. 16 - Horizontal Plane:** maximum movement of an are of circumference with the center in the elbow and the radius = forearm hand

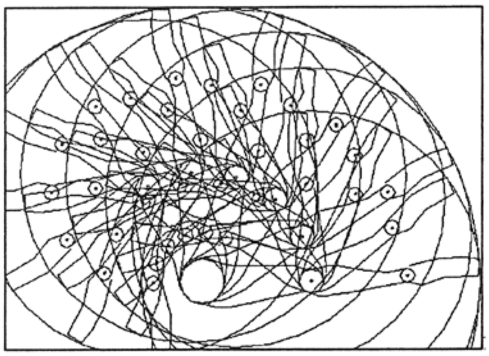


**Fig. 19 - Detail of the double movement.**

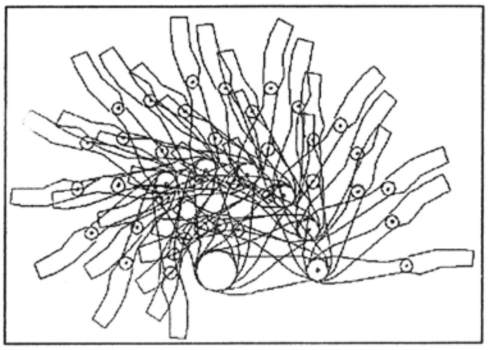


**Fig. 17 - Horizontal Plane:** two maximum movement at the same time. Arc with the center in the shoulder and are with the center in the elbow.

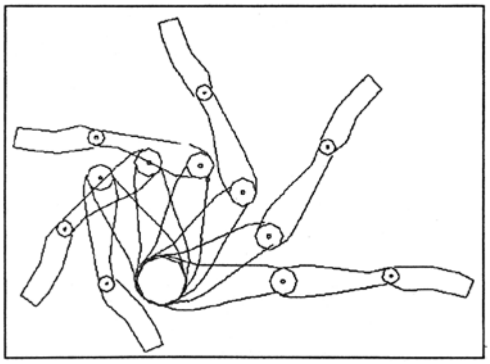


**Fig. 20 - Horizontal Plane:** six sequential position of a maximum triple movement: Flexibility in the arm, elbow and wrist.

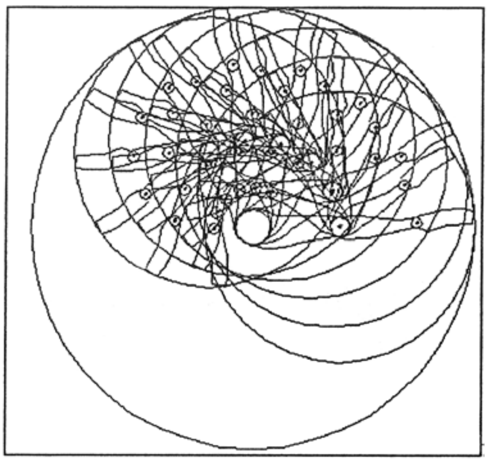


**Fig. 18 - Horizontal Plane:** the sequence of movement with the center in the elbow describes a series of cincumferences internally tangent to a circumference whose center is the shoulder and the radius is the length from the shoulder to the extremity of the middle finger.

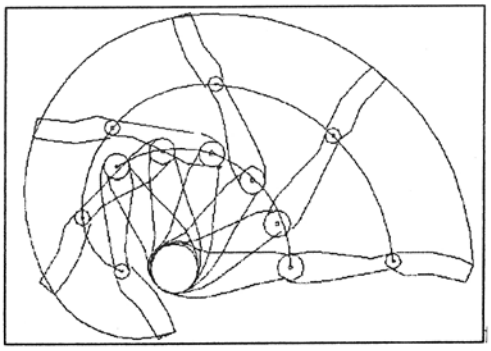


**Fig. 21 - Horizontal Plane:** Point trajectories in space: Extremy of the middle finger: curve of infinite center. Wrist: curve of infinite centers. Elbow: arc of circumferences.

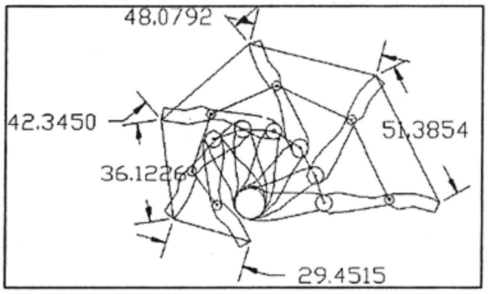



**Fig. 22 - Horizontal Plane:** Sequence of different distances between the points, described within a same period.

**Observations:** Horizontal Plane: relation among distance, time and angles decribed by the maximum movement of the stretched arm, projected in rel greatness on the horizontal plane.
The distances and the angles among the 6 positions are progressively reduced from the maximum stretched position to the maximum position of flexibility

**Distance Variation:**
62.4741 - 3.1951 = 59.2790
59.2790 - 7.2239 = 52.0551
52.0551 - 7.6127 = 44.4424
44.4424 - 8.5043 = 35.9381
35.9381

**Angle Variations:**
121 degrees - 1 degree = 120 degrees
120 degrees - 3 degree = 117 degrees
117 degrees - 5 degree = 112 degrees
112 degrees

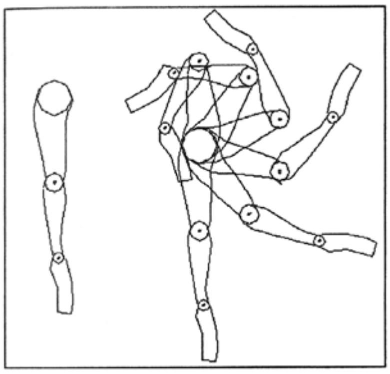


**Fig. 23 - Lateral Plane:** The maximum triple movement with three points of flexibility: shoulder, elbow and wrist.

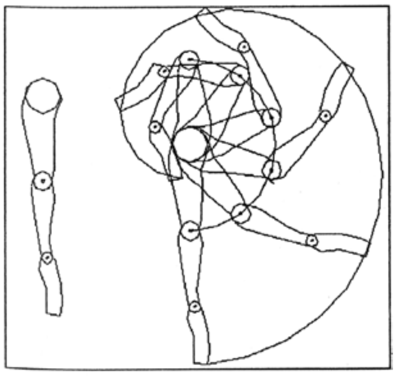


**Fig. 24 - Lateral Plane:** The points trajectory in space during a triple movement.
The extremity of the middle finger: curve of infinite centers.
Wrist: curve of infinite centers.
Elbow: arc of circumference.

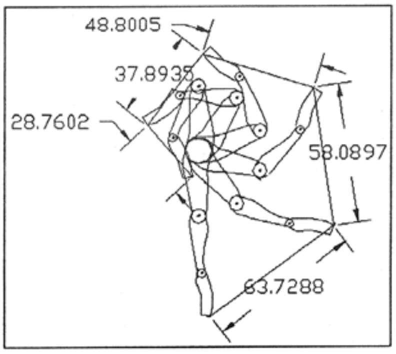



**Fig. 25 - Lateral Plane:** The relationship between the different distances determined by the continuous movement of a point within the same period.

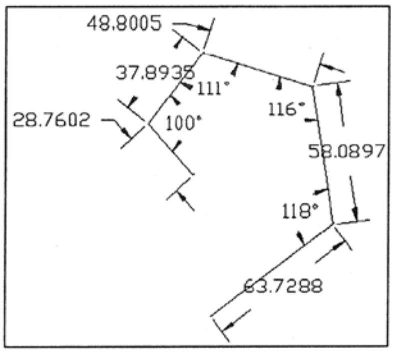



**Fig. 26 - Lateral Plane:** The relationship between the distances and angles described by a point in continuous movement within the same period.

**Observations:** The distances and the angies among the 6 positions are progressively reduced from the maximum stretched position to the fold position of maximum flexibility.

**Dimensions Variations:**
63.7288 - **05 6383** = 58 0897
58.0897 - **09 2892** = 48 8005
48.8005 - **10 9070** = 37 8935
37.8935 - **09 1333** = 28 7602
28.7602

**Angle Variantions:**
118 degrees - **2 degrees** = 116 degrees
116 degrees - **5 degrees** = 111 degrees
111 degrees - **11 degrees** = 100 degrees
100 degrees

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